# Comparison of the moore’s method and dijkstra’s algorithm in determing the shortest path between two points on a map

BY GROUP 18
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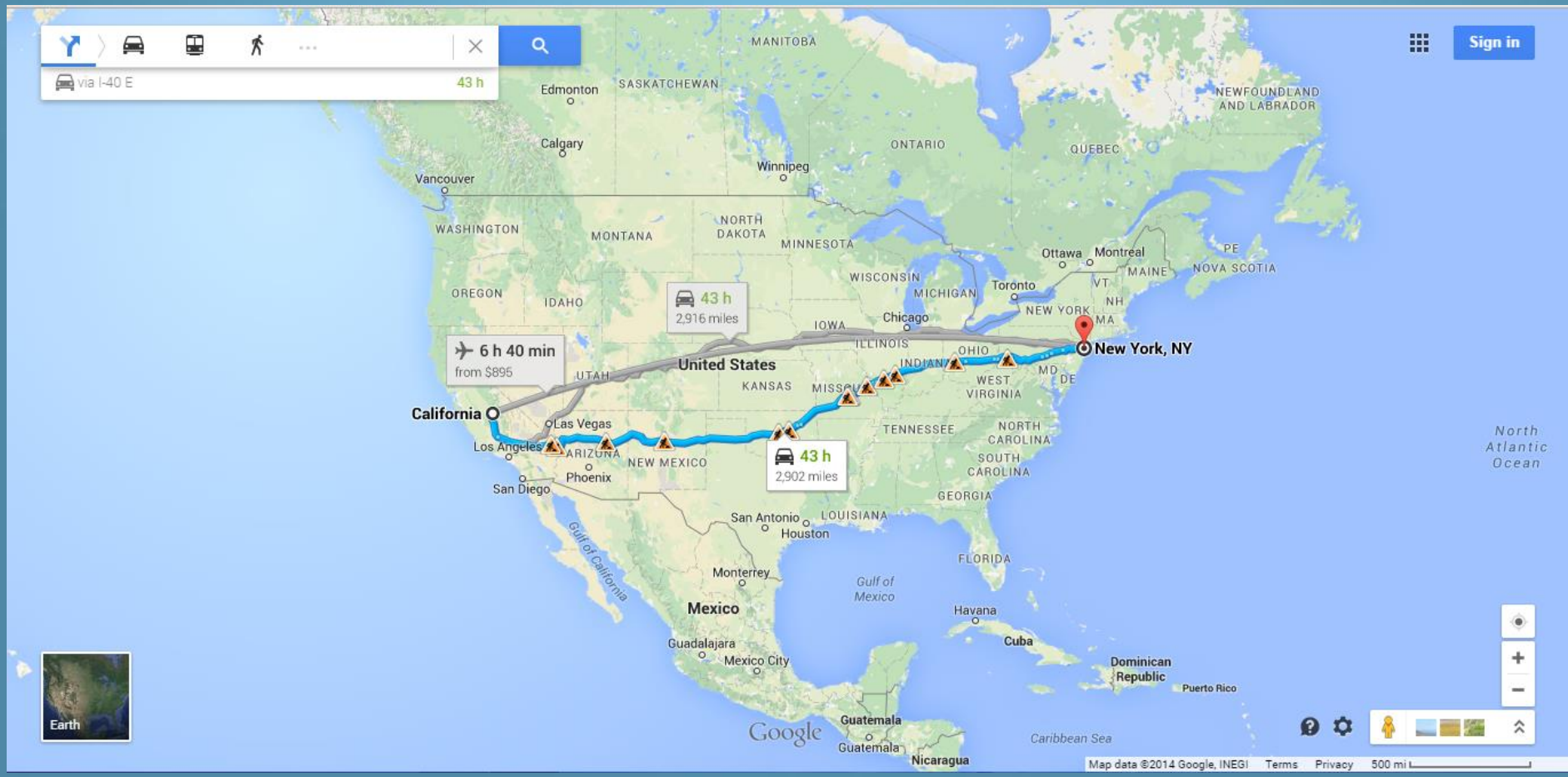
# MOTIVATION

- Global Positioning System (GPS) navigation onto  smartphone
- Limitations of processing capability
- More efficient algorithm necessary

# MOTIVATION


via I-40 E
43 h
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6 h 40 min
from $895
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New York, NY
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Las Vegas
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Phoenix
San Diego
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Toronto
Ottawa
Montreal
Vancouver
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Edmonton
Winnipeg
Houston
San Antonio
Monterrey
Mexico
Guadalajara
Mexico City
Havana
Cuba
Dominican Republic
Puerto Rico
Guatemala
Nicaragua
Gulf of Mexico
Caribbean Sea
North Atlantic Ocean
Earth
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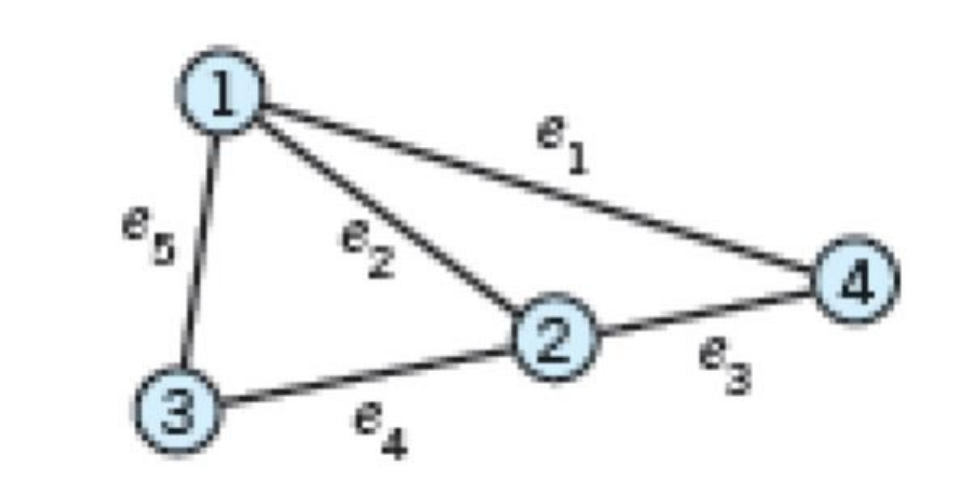
# Problem description

- Maps made with nxn number of tiles
- A starting point and ending point would be determined within this map
- Maps can be represented as graphs
- Compare Moore's and Dijkstras method for these graphs

# Introduction to methods

## Background Knowledge




- A set V of vertices: stand for spots in map models
- A set E of edges: stand for routes connecting spots

# Moore’s Method (Breadth-First-Search Algorithm)

- The widely used fundamental method in searching for the shortest path
- Blind searching method: searching every node of a map to find out the shortest path

# Procedure of the BFS method

1.Label the start point s with 0;

2.Set i=0;

3.Search all unlabeled vertices adjacent to a vertex labeled i;

4.Label all vertices just found with i+1;

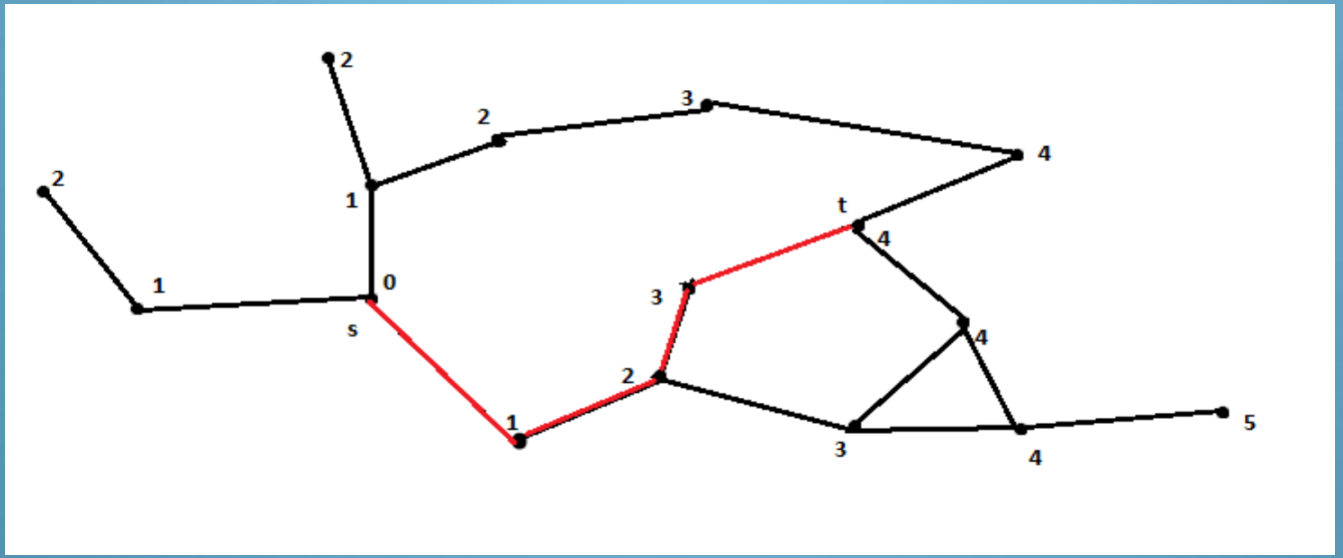
5.If the terminal point t is found, stop the process and output the shortest path. Otherwise, loop back to the step 3 and keep going on.

# IMPORTANT ASSUMPTION

- Every edge between any two vertices has the same length.

- Problem changed from finding out the shortest path to finding out the path with the smallest number of edges.

# EXAMPLE




Solution:

1.Label the start point s with 0;

2.Search all unlabeled vertices adjacent to the point 0 and label them with 1;

3.Repeat the previous works to vertices with new labels, until getting the terminal point t labeled;

4.The number that terminal point labeled with is the shortest length, and the red path is the shortest

# Dijkstra's Algorithm

- Kind of similar to the BFS method

- Each vertex is labeled in either two ways

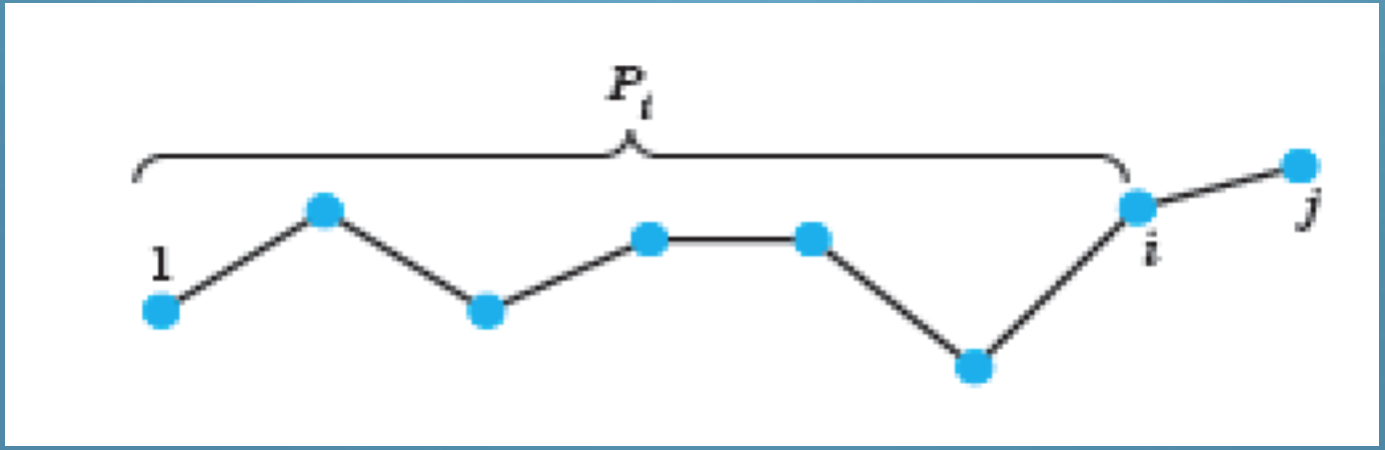
1.A permanent label $L_j$ for the length of a shortest path $1 \rightarrow j$;

2.A temporary label with stared $L_j^*$ for the length of a shortest path $1 \rightarrow j$;

3.Set up two sets(stacks) of ***ps*** and ***ts*** respectively for the permanent labels and temporary ones.

# Bellman's Optimality Principle

If **P**: **1→ j** is a shortest path from 1 to **j** in graph ***G*** and **( i, j)** is the last edge of $\mathbf{P}_j$ ( as picture following), then $\mathbf{P}_i$ **:  1→ j** [ obtained by dropping ( i, j) from $P_j$] is a shortest path **1→ j.**



# Pseudcode

**Input**: Number of vertices **n**, edges **( i, j ),** and length $l_{ij}$

**Output:** Lengths $L_j$ of the shortest path **1→ j, j = 2, 3, …, n**

1.Initial Step

Vertex 1gets PL: $L_1=0$

Vertex **j (= 2,…,n)** gets TL: $L_j^*=l_{1j}$ (=∞ if there is no edge **( i, j )** in ***G***).

Set ***ps***={1}, ***ts***={2,3,…,n}.

2.Fixing a permanent label

Find a **k** in ***ts*** for which $L_k^*$ is minimum, set $L_k= L_k^*$. Take the smallest ***k*** if there are several. Delete **k** from ***ts*** and include it in ***ps***.

If ***ts*** $=\emptyset$ (that is, ***ts*** is empty) then

OUTPUT $L_2,\ldots,L_n$, stop

Else continue (that is, go to Step 3).

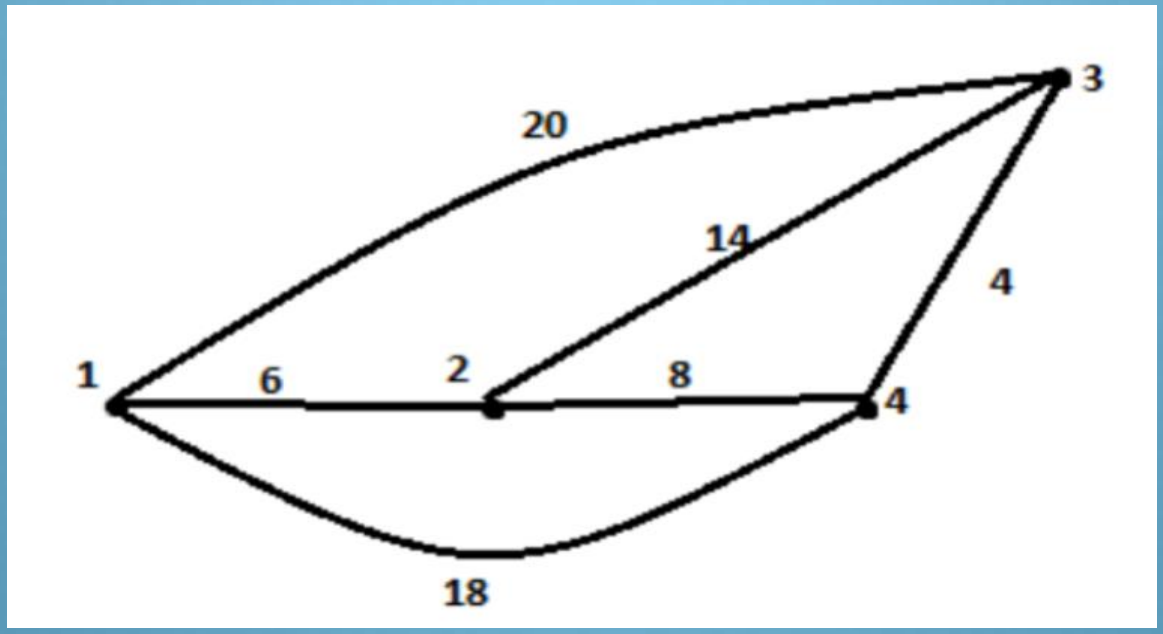
3.Updating temporary labels

For all j in ***ts***, set $L_j^* = \min \{ L_j^* , L_k + l_{kj}\}$

Go to Step 2

END

# Example




Question: find out the shortest path 1→2，3，4

**Solution**. The steps and computations.

Step 1. $L_1$=0, $L_2$*= 6, $L_3$*=20, $L_4$*=18 — ***ps*** ={1}, ***ts***={2，3，4}

Step 2. $L_2$ = min {$L_2$*, $L_3$*, $L_4$*}= 6, k=2 — ***ps*** ={1,2}, ***ts***={3，4}

Step 3. $L_3$*=min {20, $L_2$ +$L_{23}$}= min {20,6+14}=20,

$L_4$*=min {18, $L_2$ +$L_{24}$}= min {18,6+8}=14,

Step 2. $L_4$ =min {$L_3$*, $L_4$*} =14,**k**=4 — ***ps*** ={1,2,4}, ***ts***={4}

Step 3. $L_3$*=min {20, $L_4$ + $l_{43}$}= min {20, 14+4}=18

Step 2. ***L3***=18, k=3 — ***ps*** ={1,2,3,4}, ***ts*** =∅

Figure 1.4 shows the resulting shortest paths, of lengths $L_2$=6, $L_3$=18, $L_4$=14

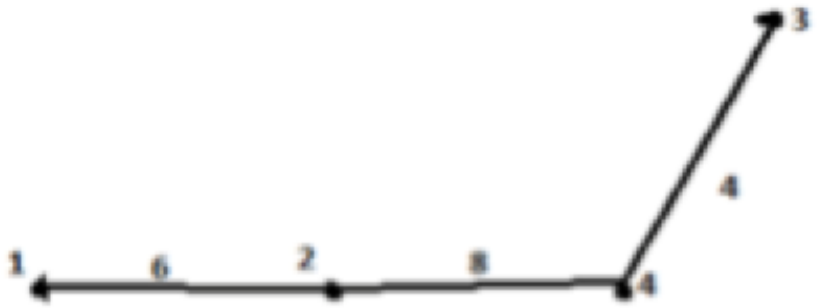



Figure 1.4 - Shortest Paths From Vertex 1 To Vertices 2, 3, 4

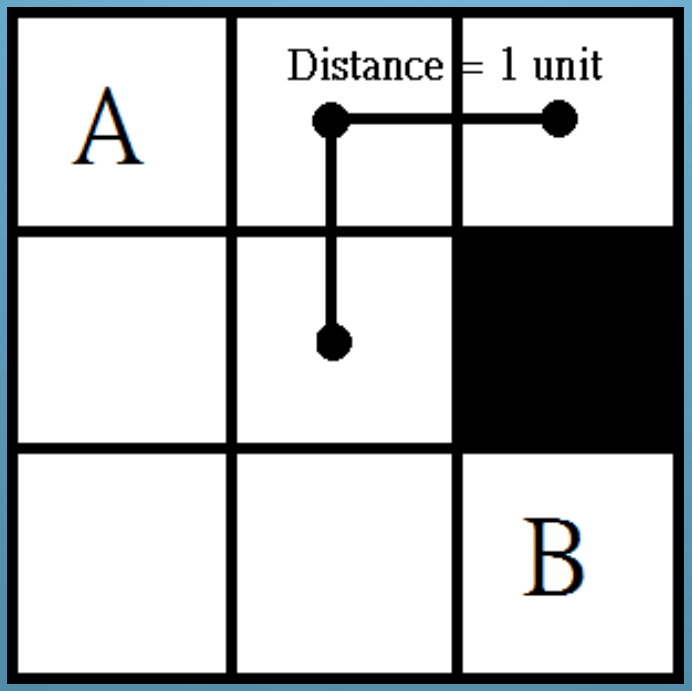
# Mathematical presentation of problem

Assumptions for this problem

- The distance between the nodes is 1 unit.
- Turing a corner has no impact on the distance travelled.
- The edge between nodes are bidirectional

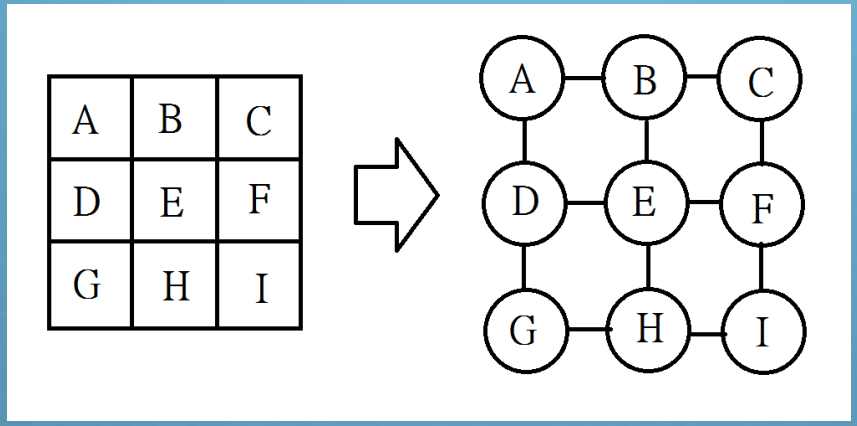
# Mathematical presentation of problem

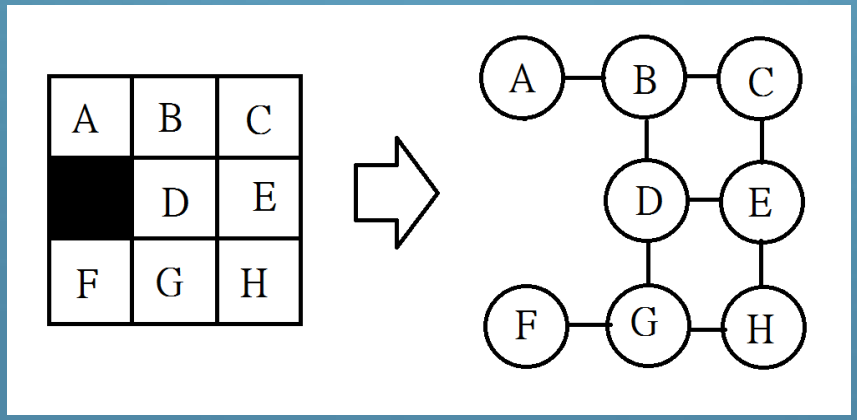
1. create maps



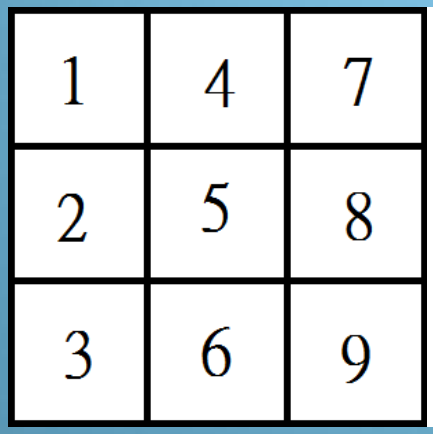
# Mathematical presentation of problem

## 2. transfer maps into graphs



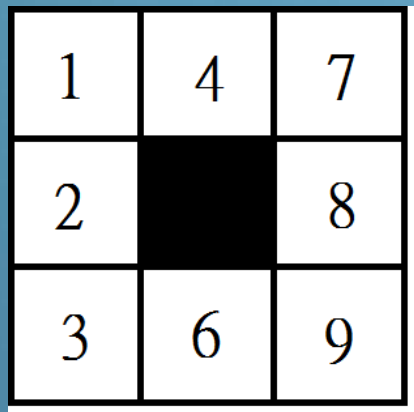



# Transforming the designed maps into graphs




node_map1 =

$$\begin{bmatrix} 1 & 2 \\ 1 & 4 \\ 2 & 3 \\ 2 & 5 \\ 3 & 6 \\ 4 & 5 \\ 4 & 7 \\ 5 & 6 \\ 5 & 8 \\ 6 & 9 \\ 7 & 8 \\ 8 & 9 \end{bmatrix}$$

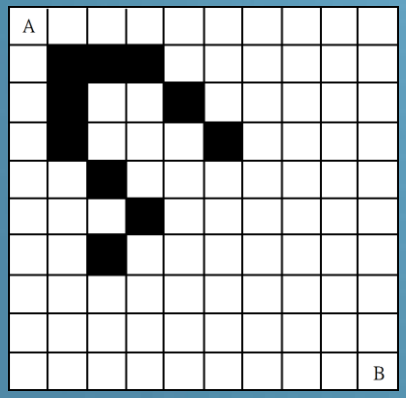



node_map1 =

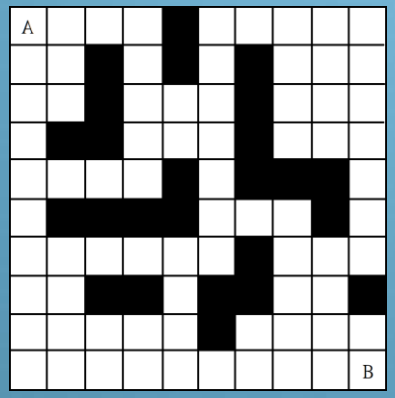
$$\begin{bmatrix} 1 & 2 \\ 1 & 4 \\ 2 & 3 \\ 3 & 6 \\ 4 & 7 \\ 6 & 9 \\ 7 & 8 \\ 8 & 9 \end{bmatrix}$$

# Mathematical presentation of problem

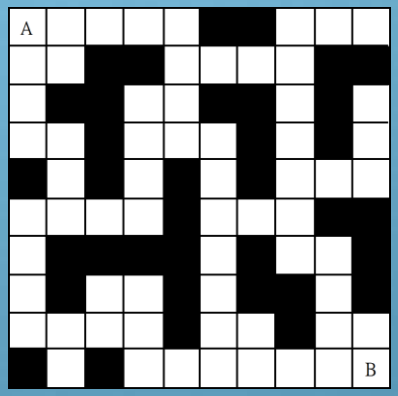
maps



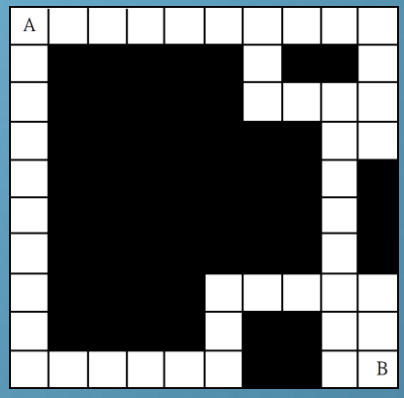

(a)




(b)



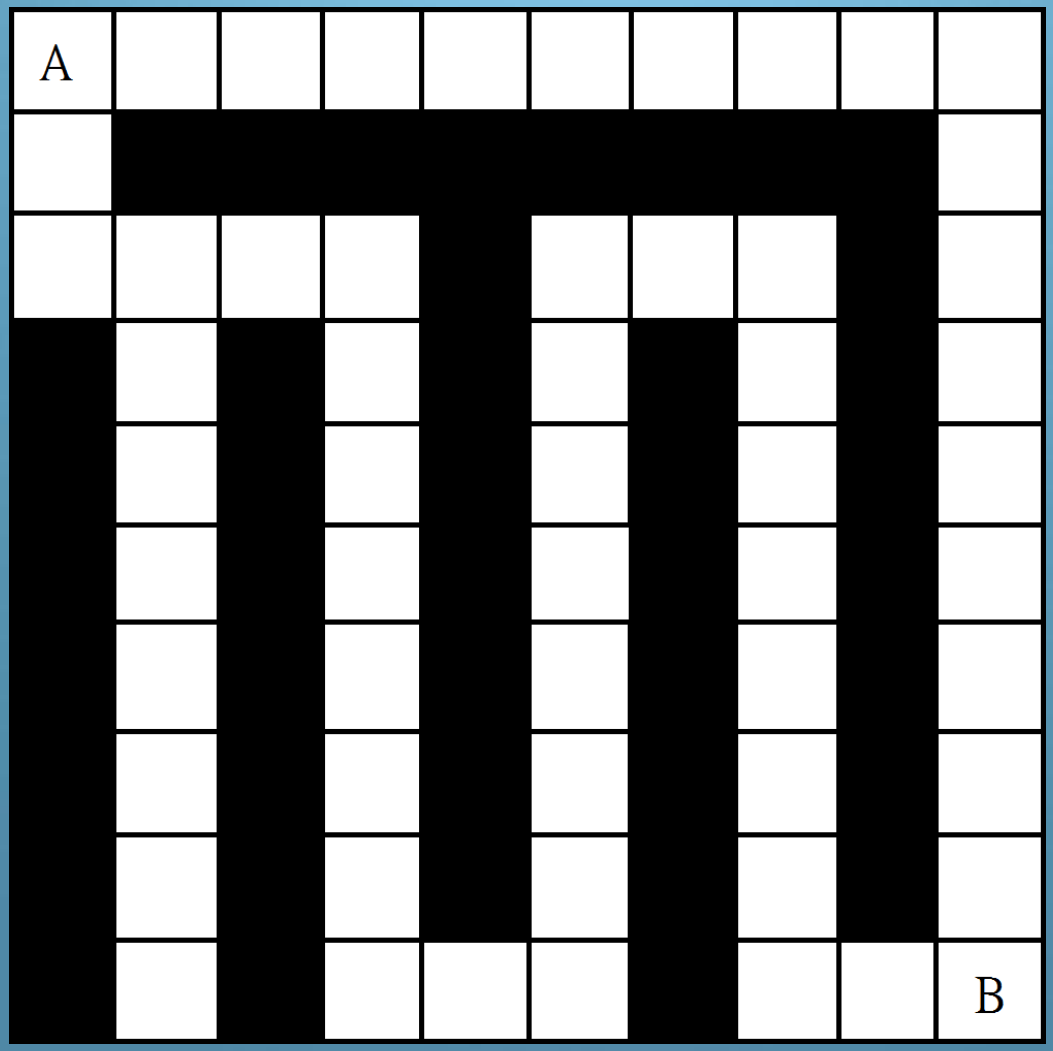

(c)




(d)

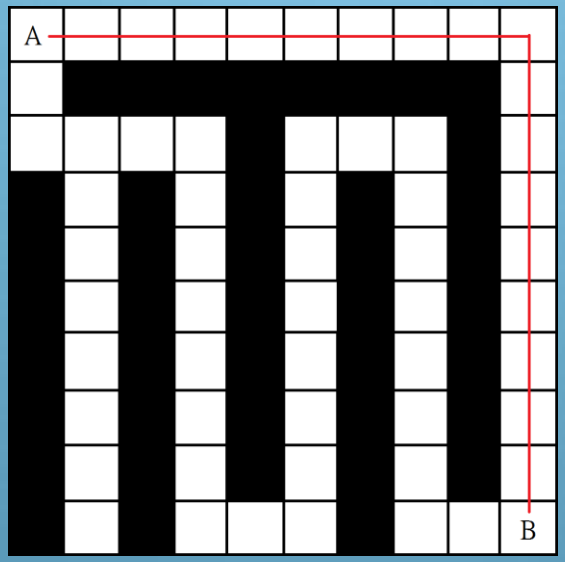
Maps (a), (b), (c), (d) have been developed with 10, 25, 36 and 55 obstacles

# Reference map



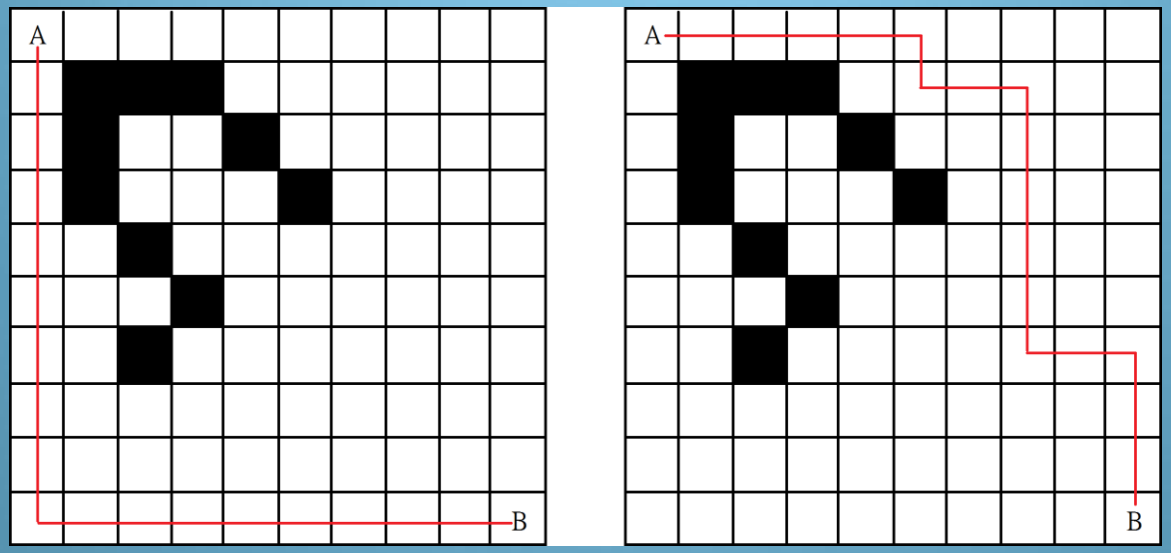
# Reference map

Upon the application of the MATLAB functions to the reference map, the same shortest path was found successfully by both methods.




| | Moore's | Dijkstra's |
|---|---|---|
| Reference Map (average time) | 0.000733s | 0.000743s |
| Shortest path (node – node) | 1-11-21-31-41-51-61-71-81-91-92-93-94-95-96-97-98-99-100(18steps) | 1-11-21-31-41-51-61-71-81-91-92-93-94-95-96-97-98-99-100(18steps) |

# Maps with different number of obstacles

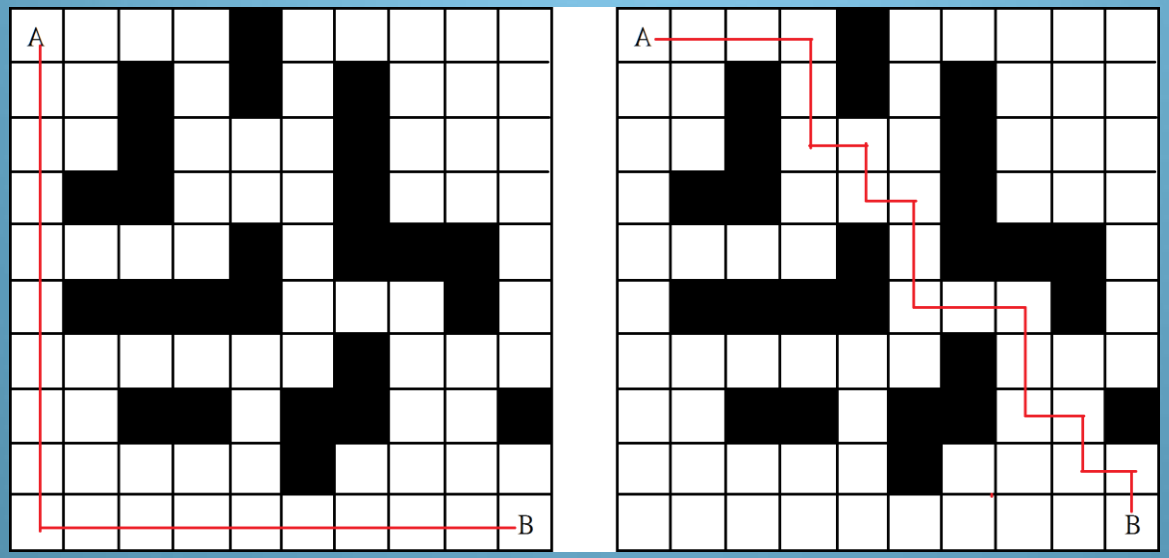



(a) (b)

(a)Moore's method (b)Dijkstra's Algorithm

| | Moore's | Dijkstra's |
|---|---|---|
| Map10 (average time) | 0.000813s | 0.000848s |
| Shortest path (node – node) | 1- 2- 3- 4- 5- 6- 7- 8- 9-10-20-30-40-50-80-90-100(18 steps) | 1-11-21-31-41-51-52-62-72-73-74-75-76-77-87-97- 98-99-100 (18 steps) |

# Maps with different number of obstacles

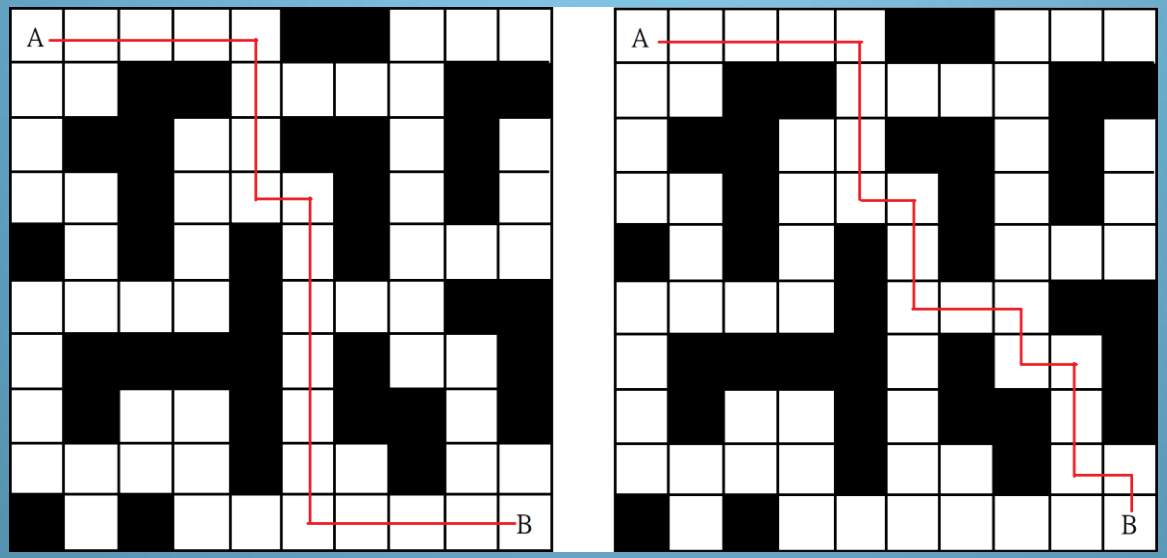



(a) (b)

(a)Moore's method (b)Dijkstra's Algorithm

| | Moore's | Dijkstra's |
|---|---|---|
| Map25 (average time) | 0.000750s | 0.000781s |
| Shortest path (node – node) | 1- 2- 3- 4- 5- 6- 7- 8- 9-10-20-30-40-50-80-90-100(18 steps) | 1-11-21-31-32-33-43-44-54-55-56-66-76-77-78-88-89-99-100 (18 steps) |

# Maps with different number of obstacles

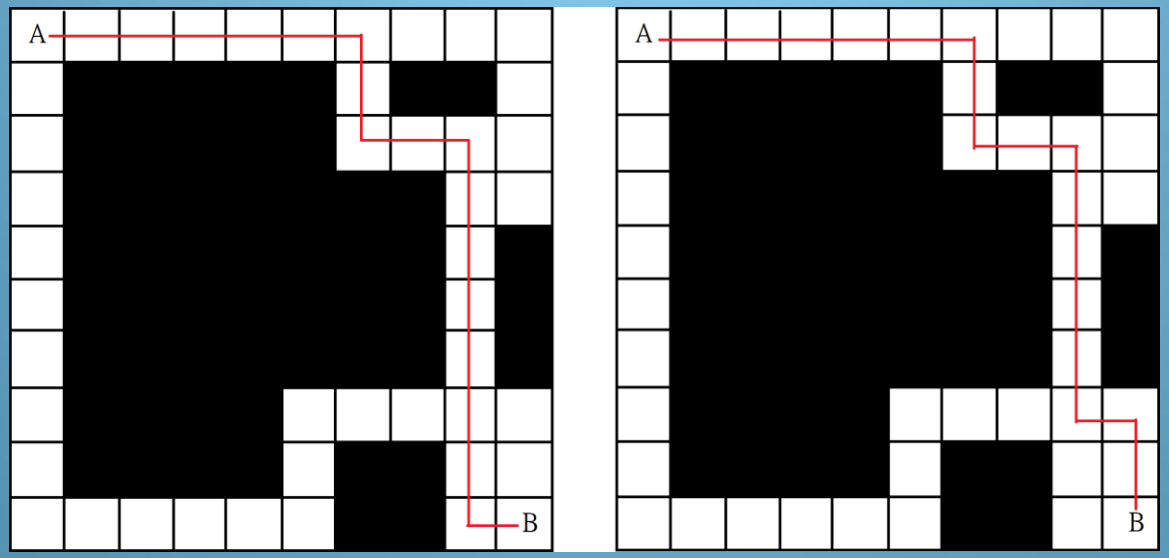



(a) (b)

(a)Moore's method (b)Dijkstra's Algorithm

| | Moore's | Dijkstra's |
|---|---|---|
| Map36 (average time) | 0.000724s | 0.000769s |
| Shortest path (node – node) | 1-11-21-31-41-42-43-44-54-55-56-57-58-59-60-70-80-90-100(18 steps) | 1-11-21-31-41-42-43-44-54-55-56-66-76-77-87-88-89-99-100(18 steps) |

# Maps with different number of obstacles

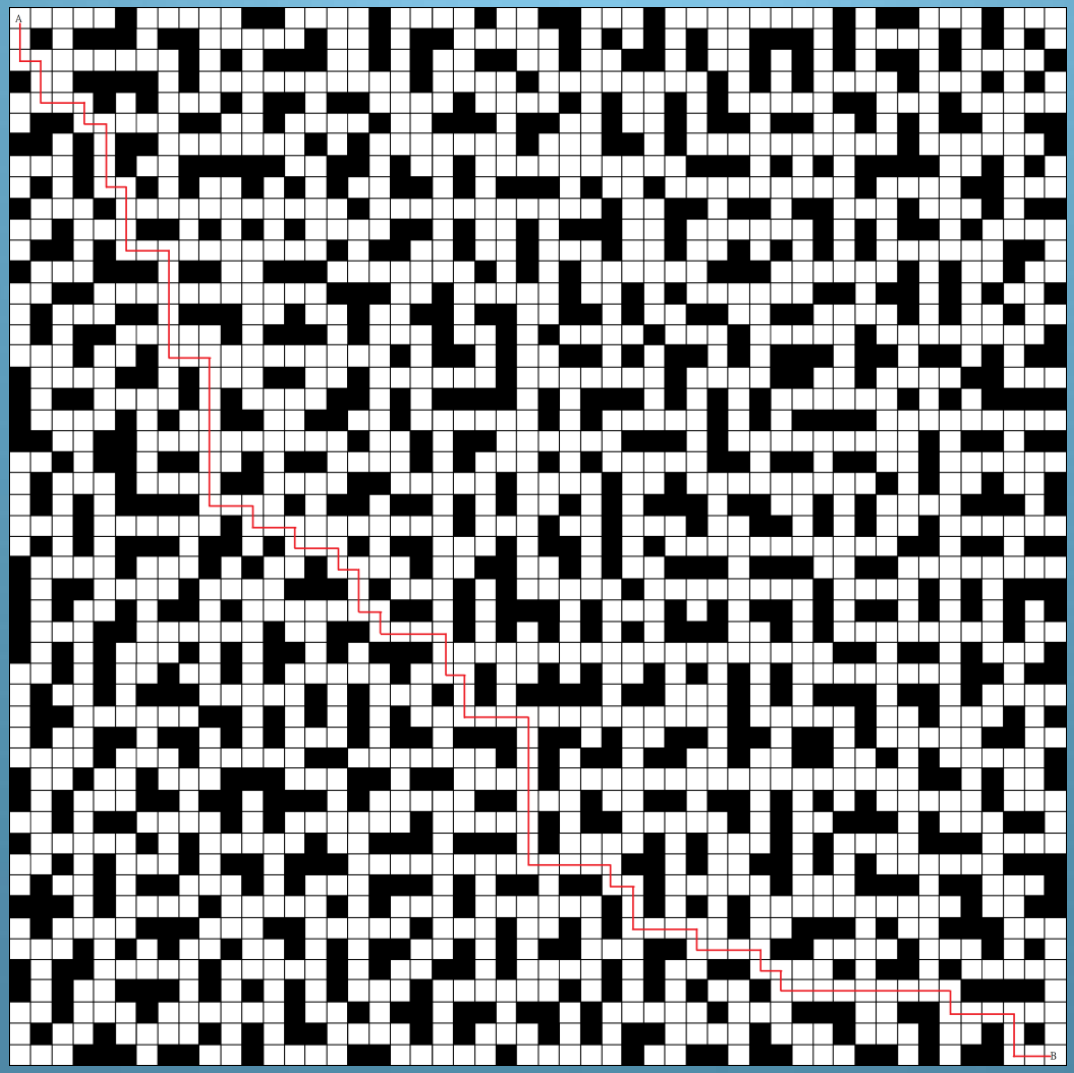



(a) (b)

(a)Moore's method (b)Dijkstra's Algorithm

| | Moore's | Dijkstra's |
|---|---|---|
| Map50 (average time) | 0.000713s | 0.000742s |
| Shortest path (node – node) | 1-11-21-31-41-51-61-62-63-73-83-84-85-86-87-88-89-90-100(18 steps) | 1-11-21-31-41-51-61-62-63-73-83-84-85-86-87-88-98-99-100(18steps) |

# 50x50 Map with 900 obstacles




the shortest path – Moore’s method

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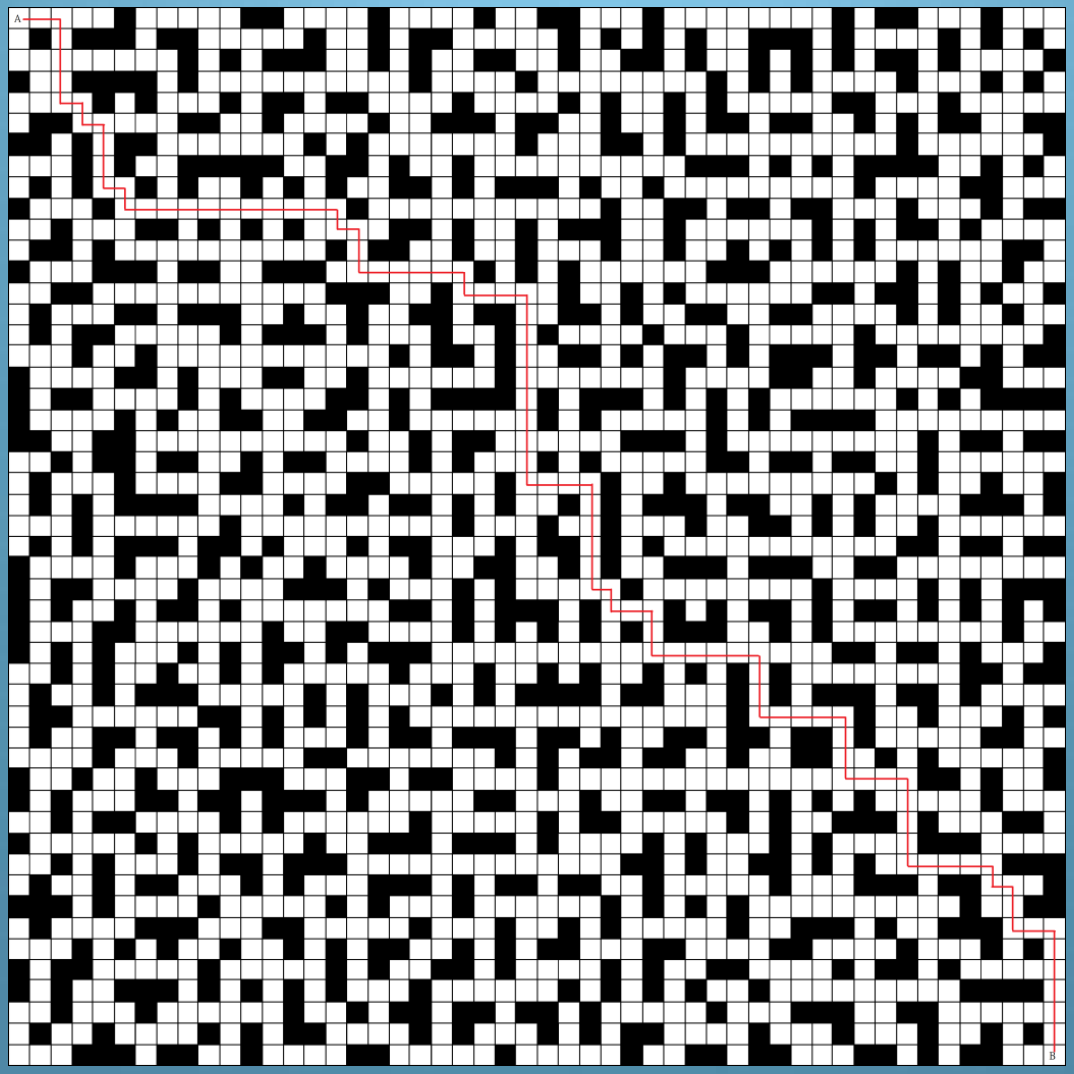


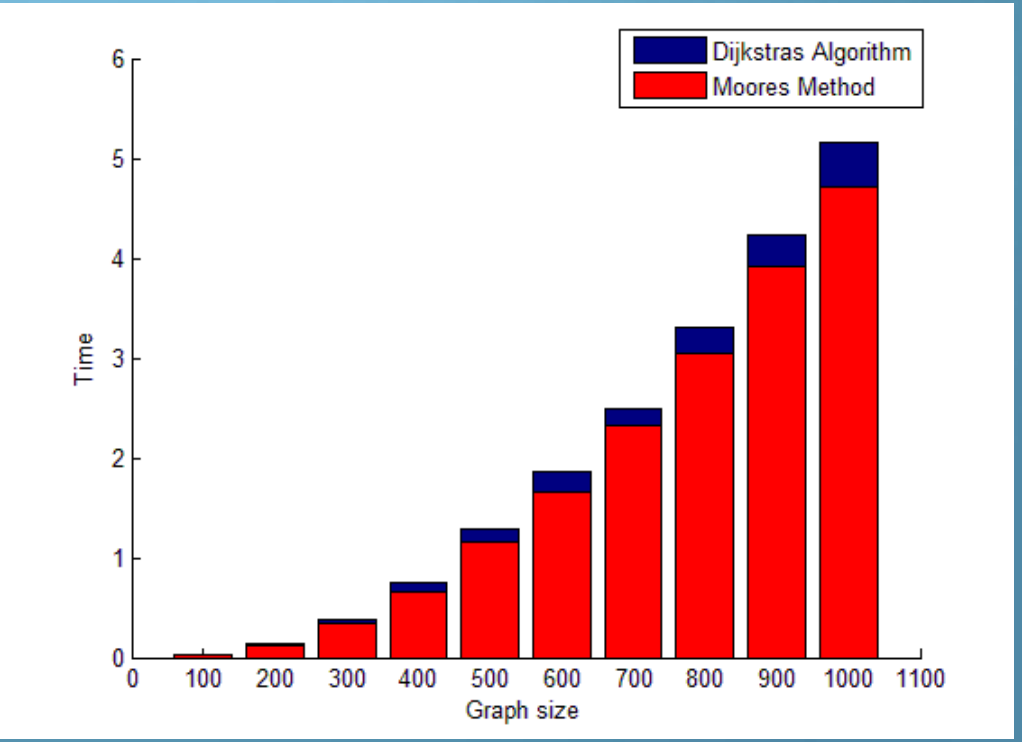

the shortest path – Moore’s method

# 50x50 Map with 900 obstacles

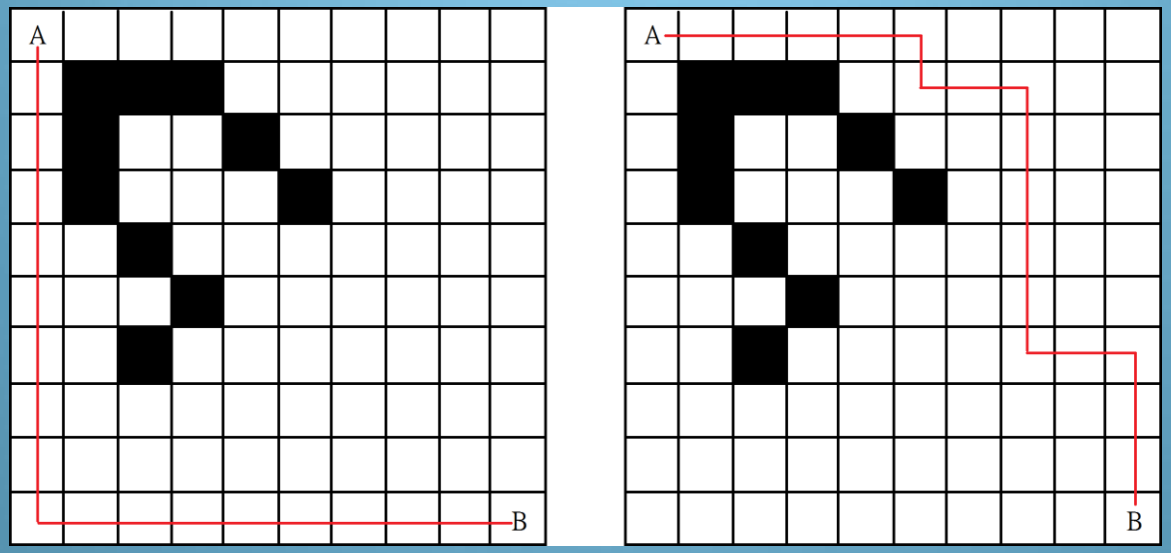
| | Moore's | Dijkstra's |
|---|---|---|
| Map 50x50 (no obstacles) | 0.006550s | 0.006604s |
| Map 50x50 (average time) | 0.00444s | 0.004850s |
| Shortest path (node – node) | 1-2-3-53-54-55-105-155-156-206-207-208-209-259-260-261-262-312-362-363-364-365-366-367-417-467-468-469-470-471-472-473-474-524-574-575-625-675-676-726-776-777-827-828-829-879-880-930-980-1030 1031-1032-1082-1083-1084-1134-1184-1234-1235-1236-1237-1238-1239-1240-1241-1291-1341-1391-1441-1442-1492-1493-1494-1544-1594-1644-1645-1695-1745-1795-1796-1846-1847-1897-1947-1997-2047-2097-2147-2197-2247-2248-2298-2348-2398-2399-2400-2450-2500 (98 steps) | 1-51-101-102-103-104-105-155-156-206-207-208-209-259-260-310-360-410-460-510-560-610-660-710-760-761-811-812-813-863-913-963-1013-1063-1064-1114-1164-1214-1215-1216-1217-1218-1219-1220-1221-1222-1223-1273-1323-1373-1374-1375-1376-1377-1378-1428-1429-1479-1529-1530-1531-1581-1631-1681-1731-1781-1782-1783-1784-1834-1884-1934-1984-1985-1986-1987-2037-2087-2137-2138-2139-2140-2141-2191-2241-2291-2341-2342-2392-2393-2394-2444-2494-2495-2496-2497-2498-2499-2500 (98 steps) |

# Timing Results

| | Moore's | Dijkstra's |
|---|---|---|
| 100x100 | 0.023097 | 0.02613 |
| 200x200 | 0.127722 | 0.141696 |
| 300x300 | 0.346553 | 0.383323 |
| 400x400 | 0.661585 | 0.756057 |
| 500x500 | 1.160261 | 1.287945 |
| 600x600 | 1.660201 | 1.852375 |
| 700x700 | 2.321567 | 2.486482 |
| 800x800 | 3.039056 | 3.296351 |
| 900x900 | 3.923256 | 4.225479 |
| 1000x1000 | 4.714555 | 5.157708 |



# Maps with different number of obstacles




(a) (b)

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|---|---|---|
| Map10 (average time) | 0.000813s | 0.000848s |
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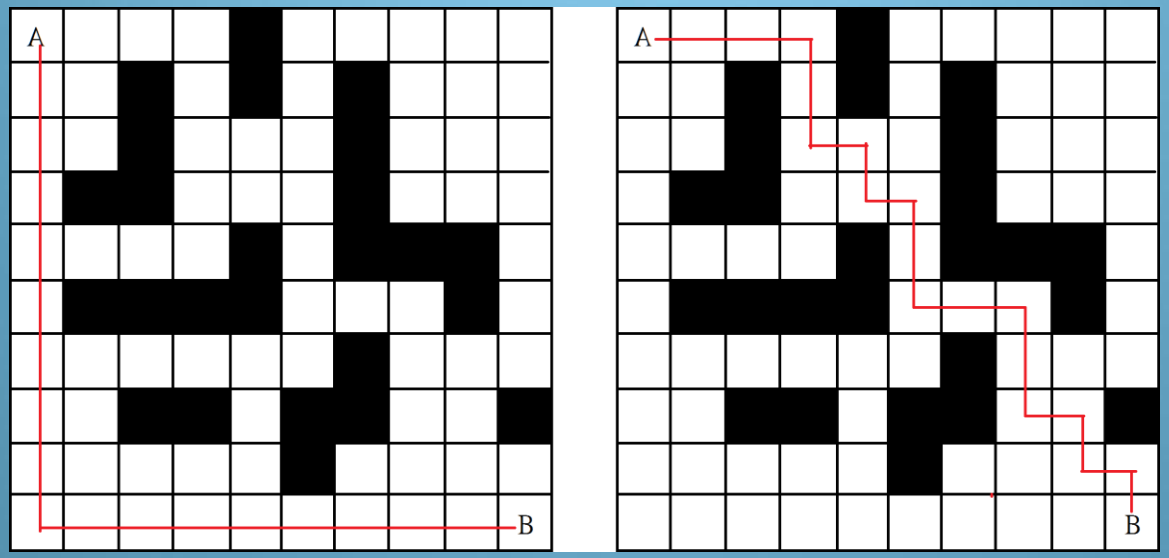
# Maps with different number of obstacles




(a) (b)

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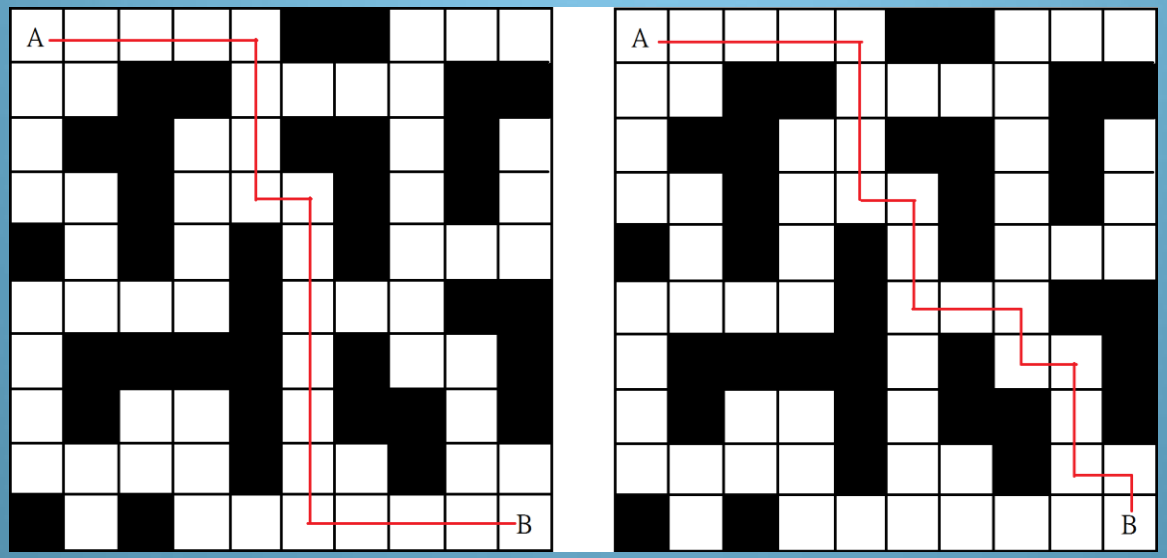


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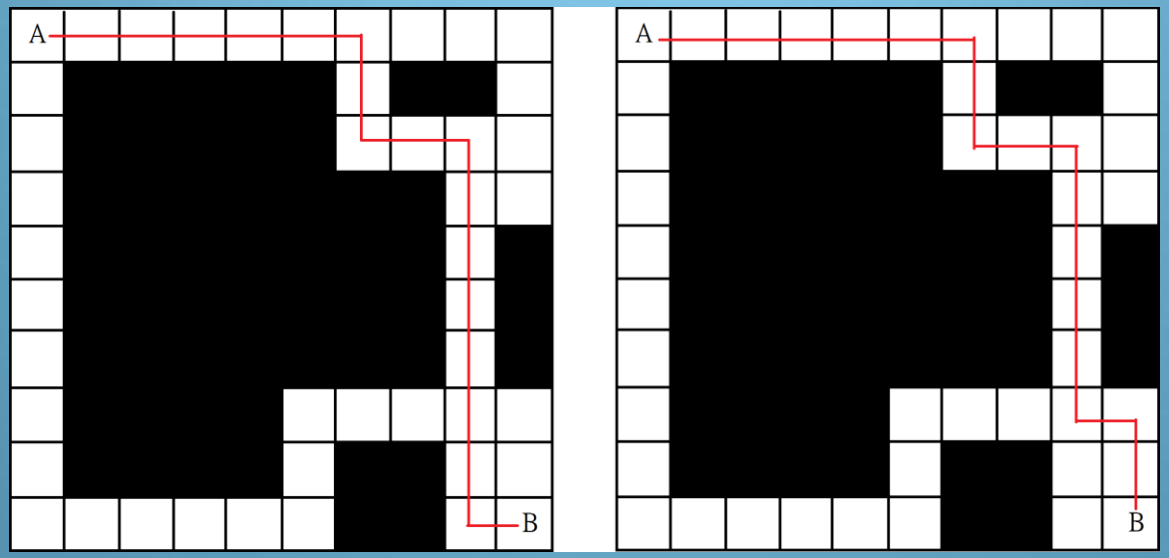
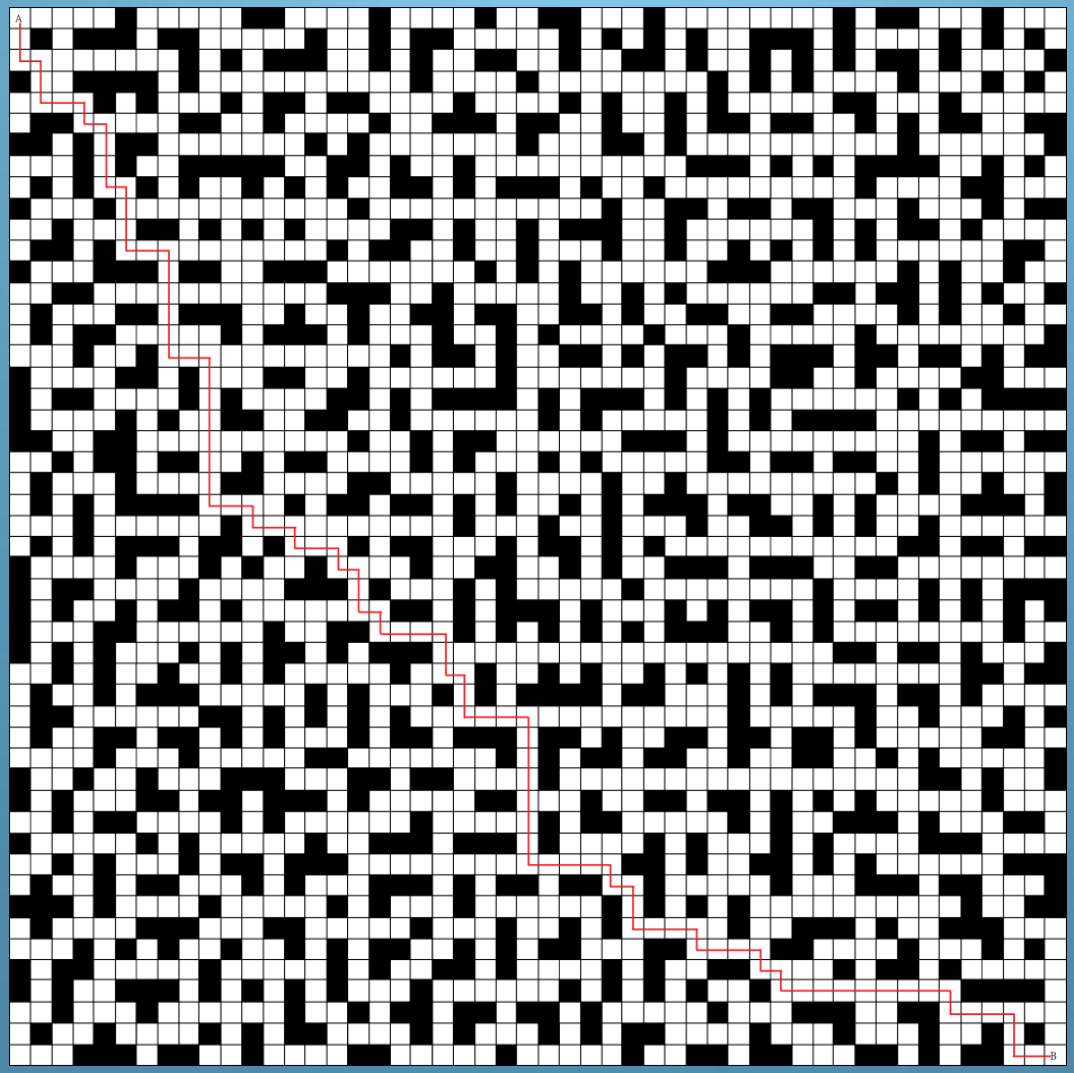
# Maps with different number of obstacles




(a) (b)

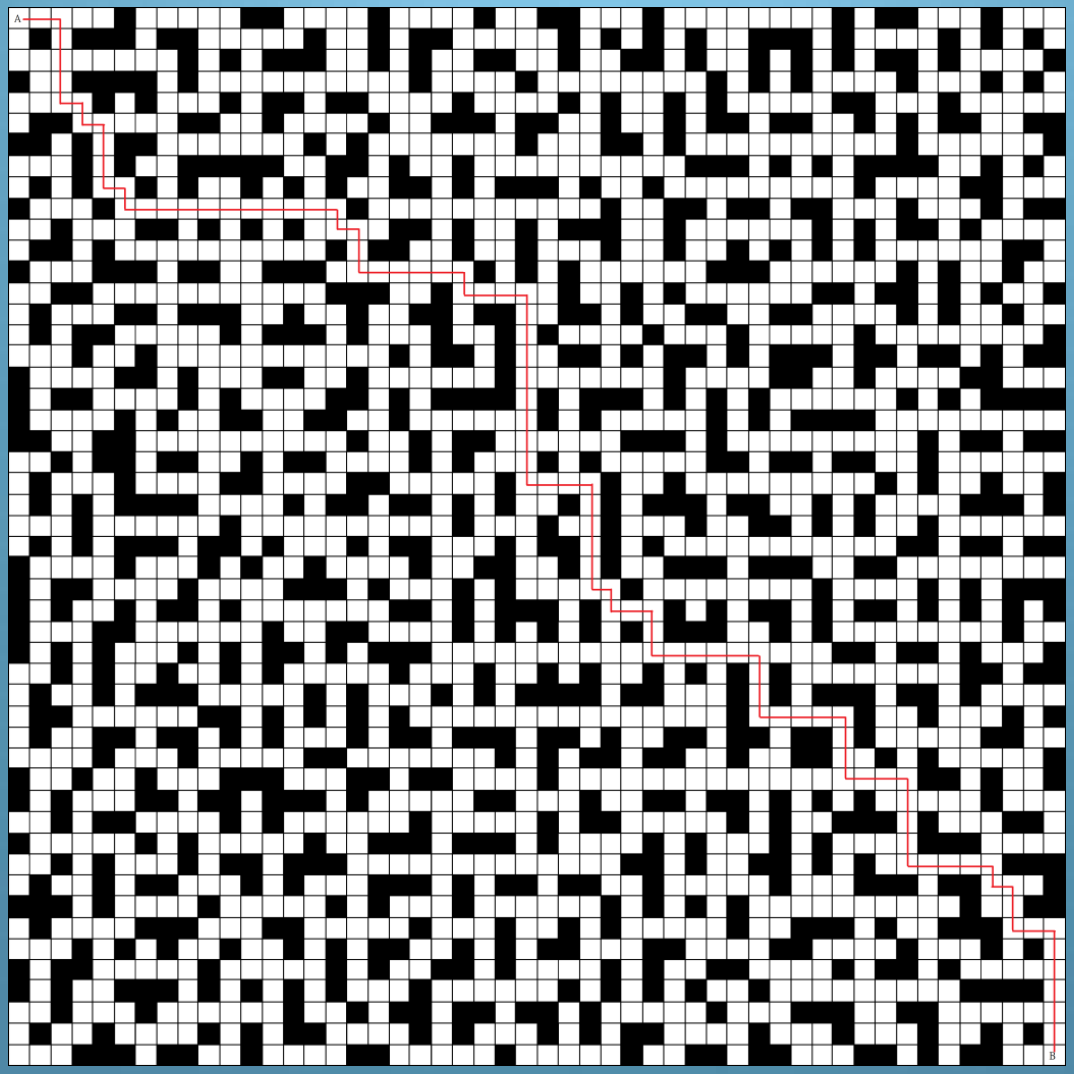
| | Moore's | Dijkstra's |
|---|---|---|
| Map50 (average time) | 0.000713s | 0.000742s |
| Shortest path (node – node) | 1-11-21-31-41-51-61-62-63-73-83-84-85-86-87-88-89-90-100(18 steps) | 1-11-21-31-41-51-61-62-63-73-83-84-85-86-87-88-98-99-100(18steps) |

# 50x50 Map with 900 obstacles




the shortest path – Moore’s method

# 50x50 Map with 900 obstacles



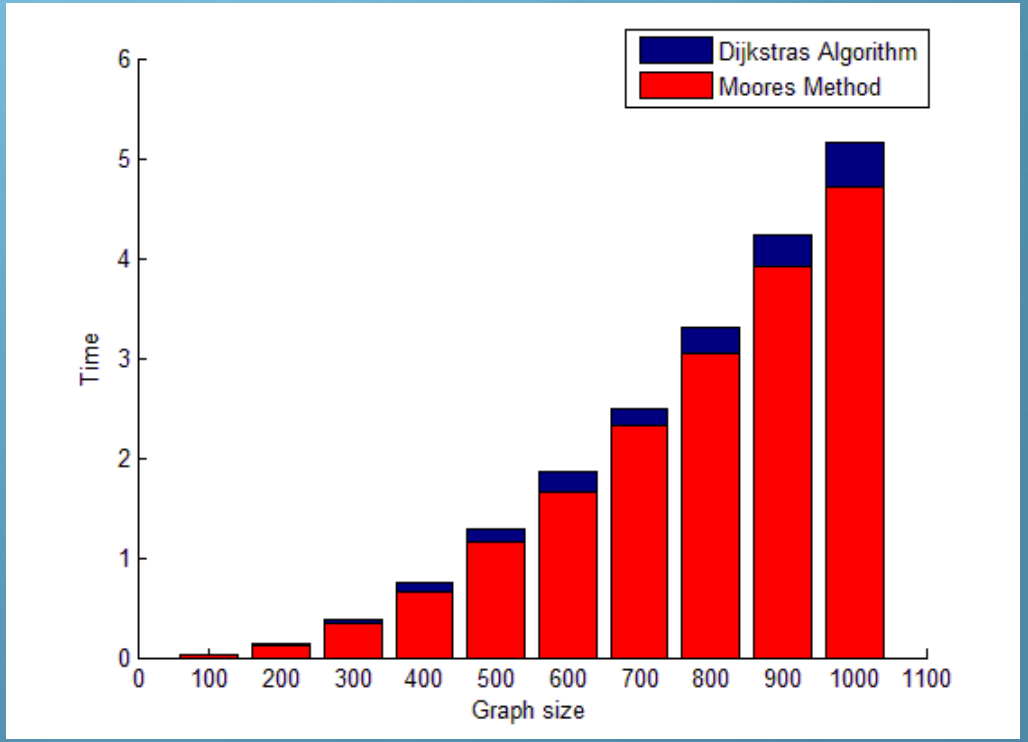

the shortest path – Moore’s method

# 50x50 Map with 900 obstacles

| | Moore's | Dijkstra's |
|---|---|---|
| Map 50x50 (no obstacles) | 0.006550s | 0.006604s |
| Map55 (average time) | 0.00444s | 0.004850s |
| Shortest path (node – node) | 1-2-3-53-54-55-105-155-156-206-207-208-209-259-260-261-262-312-362-363-364-365-366-367-417-467-468-469-470-471-472-473-474-524-574-575-625-675-676-726-776-777-827-828-829-879-880-930-980-1030 1031-1032-1082-1083-1084-1134-1184-1234-1235-1236-1237-1238-1239-1240-1241-1291-1341-1391-1441-1442-1492-1493-1494-1544-1594-1644-1645-1695-1745-1795-1796-1846-1847-1897-1947-1997-2047-2097-2147-2197-2247-2248-2298-2348-2398-2399-2400-2450-2500 (98 steps) | 1-51-101-102-103-104-105-155-156-206-207-208-209-259-260-310-360-410-460-510-560-610-660-710-760-761-811-812-813-863-913-963-1013-1063-1064-1114-1164-1214-1215-1216-1217-1218-1219-1220-1221-1222-1223-1273-1323-1373-1374-1375-1376-1377-1378-1428-1429-1479-1529-1530-1531-1581-1631-1681-1731-1781-1782-1783-1784-1834-1884-1934-1984-1985-1986-1987-2037-2087-2137-2138-2139-2140-2141-2191-2241-2291-2341-2342-2392-2393-2394-2444-2494-2495-2496-2497-2498-2499-2500 (98 steps) |

# Timing Results

| | Moore's | Dijkstra's |
|---|---|---|
| 100x100 | 0.023097 | 0.02613 |
| 200x200 | 0.127722 | 0.141696 |
| 300x300 | 0.346553 | 0.383323 |
| 400x400 | 0.661585 | 0.756057 |
| 500x500 | 1.160261 | 1.287945 |
| 600x600 | 1.660201 | 1.852375 |
| 700x700 | 2.321567 | 2.486482 |
| 800x800 | 3.039056 | 3.296351 |
| 900x900 | 3.923256 | 4.225479 |
| 1000x1000 | 4.714555 | 5.157708 |



# Conclusion

- The Moore's method is more time efficient than the Dijkstra's algorithm for  the applications in this project

- As the map gets larger, the time required for both methods take longer

- The Dijkstra's algorithm would be more suitable for analysing weighted graphs

# Future work

- Improve MATLAB algorithm to produce weighted maps for Dijkstra's algorithm

- Investigate the effect of adding distance weighting to corners